NAG Toolbox for MATLAB

s17ad

1 Purpose

s17ad returns the value of the Bessel Function $Y_1(x)$, via the function name.

2 Syntax

```
[result, ifail] = s17ad(x)
```

3 Description

s17ad evaluates an approximation to the Bessel Function of the second kind $Y_1(x)$.

Note: $Y_1(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on four Chebyshev expansions:

For $0 < x \leq 8$,

$$Y_1(x) = \frac{2}{\pi}\ln x\frac{x}{8}\sum_{r=0}{}' a_r T_r(t) - \frac{2}{\pi x} + \frac{x}{8}\sum_{r=0}{}' b_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$Y_1(x) = \sqrt{\frac{2}{\pi x}}\left\{P_1(x)\sin\left(x - 3\frac{\pi}{4}\right) + Q_1(x)\cos\left(x - 3\frac{\pi}{4}\right)\right\}$$

where $P_1(x) = \sum_{r=0}{}' c_r T_r(t)$,

and $Q_1(x) = \frac{8}{x}\sum_{r=0}{}' d_r T_r(t)$, with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $Y_1(x) \simeq -\frac{2}{\pi x}$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***. For extremely small x, there is a danger of overflow in calculating $-\frac{2}{\pi x}$ and for such arguments the function will fail.

For very large x, it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_1(x)$; only the amplitude, $\sqrt{\frac{2}{\pi x}}$, can be determined and this is returned on soft failure. The range for which this occurs is roughly related to ***machine precision***; the function will fail if $x \gtrsim 1/$***machine precision***.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W 1962 Mathematical tables *Chebyshev Series for Mathematical Functions* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $\mathbf{x} > 0.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

x is too large. On soft failure the function returns the amplitude of the Y_1 oscillation, $\sqrt{\frac{2}{\pi x}}$.

ifail $= 2$

$\mathbf{x} \leq 0.0$, Y_1 is undefined. On soft failure the function returns zero.

ifail $= 3$

x is too close to zero, there is a danger of overflow. On soft failure, the function returns the value of $Y_1(x)$ at the smallest valid argument.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_1(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x.)

If δ is somewhat larger than the ***machine precision*** (e.g., if δ is due to data errors etc.), then E and δ are approximately related by:

$$E \simeq |xY_0(x) - Y_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xY_0(x) - Y_1(x)|$.

However, if δ is of the same order as ***machine precision***, then rounding errors could make E slightly larger than the above relation predicts.

For very small x, absolute error becomes large, but the relative error in the result is of the same order as δ.

For very large x, the above relation ceases to apply. In this region, $Y_1(x) \simeq \frac{2}{\pi x} \sin\left(x - \frac{3\pi}{4}\right)$. The amplitude $\frac{2}{\pi x}$ can be calculated with reasonable accuracy for all x, but $\sin\left(x - \frac{3\pi}{4}\right)$ cannot. If $x - \frac{3\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin\left(x - \frac{3\pi}{4}\right)$ is determined by θ only. If $x > \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of, the inverse of the ***machine precision***, it is impossible to calculate the phase of $Y_1(x)$ and the function must fail.

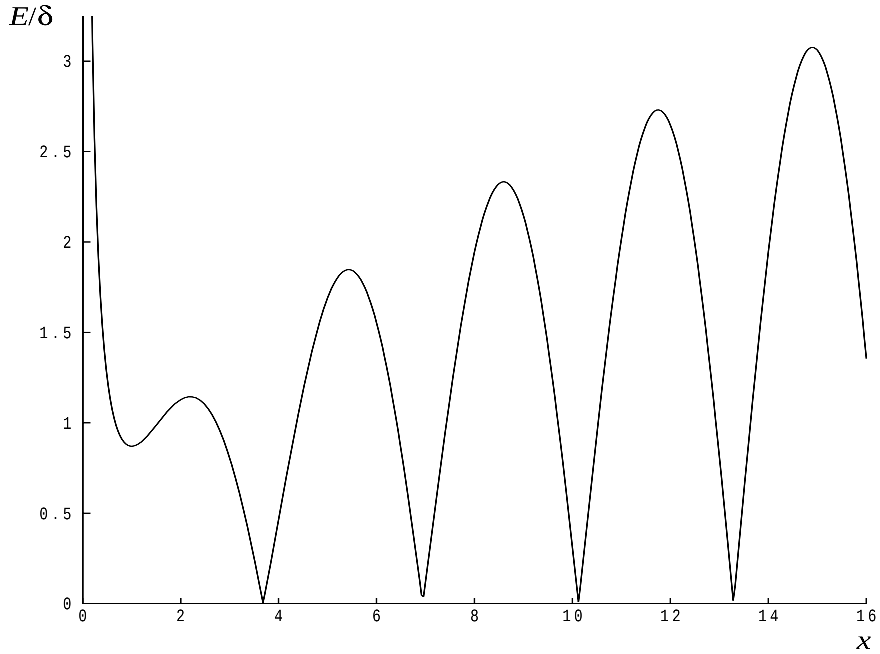

Figure 1

8 Further Comments

None.

9 Example

```
x = 1;
[result, ifail] = s17ad(x)
```

```
result =
   -0.7812
ifail =
           0
```